

# FREGE/ON A CONTEMPORARY USAGE OF FREGE ALAIN BADIOU

*[Here follows an excerpt from *Le Nombre et les nombres* by Alain Badiou, in which he lays out the axioms of set theory on which he bases his bold claim that being — what is — is pure multiplicity. The lucid style of writing does not gainsay the fact that the core idea — of an ontology that would be mathematized and thus freed from the thanato-theology that ordinarily corrupts it — is a difficult one to grasp and requires considerable patience. Since it is impossible in a short space to do justice to the comprehensiveness of Badiou's fine argument, we have chosen this excerpt with care, believing it gives the reader at least a sense of that argument's stakes. For these pages concern a dramatic moment in mathematical theory precipitated by Russell's devastating letter to Frege, who recognized immediately its consequences for his life's work. What Frege neglected to consider — and Russell's tendering of a "paradoxical" set made plain — is that not every concept has a definite extension; in some cases a predicate or property cannot define a set, a totality. Outside the confines of Zermelo's axiom of separation, that is, in the actual world, one cannot suppose an "all there is," a universe of things. The world is, in other words, actually infinite.]*

To this mathematical drama, this excerpt might be viewed as adding a cinematic one. Badiou appends to this discussion a note on Jacques-Alain Miller's concept of suture as it was proposed in a paper delivered to Lacan's seminar in 1965 and titled "Suture (elements of the logic of the signifier)." This dazzling paper, first published in French in 1966, was later translated by Jacqueline Rose and printed in *Screen* vol. 18, no. 4 (Winter 1977/78) in the now infamous "Dossier on Suture." Miller invented the concept of suture for psychoanalysis, to name the relation of the subject to the chain of its discourse; cinematic theorists (Jean-Pierre Oudart, Daniel Dayan, William Rothman, initially) appropriated the concept to designate a style of editing by which the diegetic field of the film knitted itself together to echo or deny (depending on the theory) an absent field, an off-screen space.

This is not the place to resurrect the often rancorous debates that surrounded the cinematic theory of suture (which even led film critics to

boycott a conference where, it was feared, suture theorists would be gathered). It must be recalled, however, that the tormenting clarity of Miller's proposition left its mark on film theory for a very long time, redirecting its very focus toward the way the film "positioned its spectator." Nor is this the moment to delve into the import of Badiou's objections to Miller's argument. We will simply point out that while Miller designates the (constitutive) empty place of reality as "subject," Badiou will name it "the event." For Badiou, then, a film text sustains a subject through fidelity to the name of a vanished event. For Miller, the subject is the contingent event that founds the text.

—Joan Copjec

## FREGE<sup>1</sup>

### 1

Frege maintains that pure thought generates number. Like Mallarmé, although without the effect of chance, Frege thinks that "all thought emits a throw of the dice."<sup>2</sup> What is called the "logicism" of Frege is very profound: number is not a singular form of Being, nor a particular property of things. It is neither empirical nor transcendental. It is no longer a constitutive category, but rather is deduced from the concept; it is, as Frege writes, a trait of the concept.<sup>3</sup>

### 2

The pivotal property that permits the transition from pure concept to number is the *extension* of the concept. How can we understand this? Given an arbitrary concept, an object "falls" under this concept if it is a "true case" of this concept, if the statement that attributes to this object the property contained in the concept is a true statement. Or, if the object validates the concept. Everything, one will notice, originates in the truth-value of statements, which is their denotation (true or false). We could hold that, if the concept generates number, this is in order that there be truth. Number is, in this sense, the index of truth, and not an index of Being.

But the idea of extension is ramified, obscure.

### 3

Let there be a concept. We will call the extension of this concept all the cases of truth (every object as a case of truth) that fall under this concept. Every concept has one extension.

Now, suppose there are two concepts, C1 and C2. We will say that they are *equinumerous* if a biunivocal correspondence exists that associates object for object everything that falls under the concept C1 and the concept C2. Thus we can define a biuni-

vocal correspondence between the extension of C1 and that of C2.

We can clearly see that Frege orients himself toward a “cardinal” definition of number, and that he does not care about the ordering structure of what falls under the concept. Biunivocity is an essential tool that is in fact characteristic of all efforts to “number” the multiple in itself, the pure multiple, subtracted from all structural considerations. To say that two concepts are equinumerous<sup>4</sup> is to say that they have the “same quantity,” that their extensions have the same extent — an abstraction made from any consideration about what the objects are that fall under these concepts.

#### 4

Number consists in *marking* equinumerosity, the quantitative identity of concepts. Whence the famous definition: “The number that belongs to the concept C is the extension of the concept ‘equinumerous to the concept C.’”<sup>5</sup> This means that every concept C generates a number, namely the set of concepts that are equinumerous to concept C, that have the “same pure quantity,” the same quantity of extension, as C. Note that a number, grasped in its being, always designates a set of concepts, being all those that validate the statement: “being a concept equinumerous to C.”

#### 5

The chain by which the concept of number is constructed is the following: Concept  $\emptyset$  Truth  $\emptyset$  Objects that fall under the concept (that validate the statement of attribution of the concept to the object)  $\emptyset$  Extension of the concept (all the truth-cases of the concept)  $\emptyset$  Equinumerosity between two concepts (by the biunivocal correspondence of their extensions)  $\emptyset$  Concepts that fall under the concept of equinumerosity to concept C (that validate the statement “being equinumerous to C”)  $\emptyset$  Extension of equinumerosity to C (set of concepts of the preceding stage)  $\emptyset$  Number that belongs to the concept (number is thus the name of the extension of equinumerosity to C).

From a simplified and operative point of view, we can also say that one departs from the concept, that one passes through the object under the condition of truth, that one thus compares concepts, and that the number names a set of concepts that have in common a possible and definite property of this comparison (equinumerosity).

#### 6

To rediscover the “usual,” familiar numbers on the basis of this pure conceptualism, that norm of truth alone, Frege begins by his admirable deduction of zero: zero is the number that belongs to the concept “not identical to itself.” Since every object is identical to itself,

the extension of the concept “not identical to itself” is empty. Zero is henceforth the set of concepts with empty extensions, and which, by this fact, are equinumerous to the concept “not identical to itself.” This means precisely that the number, which belongs to every concept whose extension is empty, is zero.

I have indicated elsewhere the passage to the number 1: “One” is the number that belongs to the concept “identical to zero.” It is interesting to note that Frege emphasizes, with regard to 1, that it has no more “intuitive” or empirical privilege than it has a transcendental foundation: “The definition of 1 has an objective legitimacy that is not subordinated to any contestation.”<sup>6</sup> Without any doubt, Frege participates in the great modern process of the destitution of the One.

The engendering of the sequence of numbers beyond 1 only poses technical problems, whose resolution, when passing from  $n$  to  $n+1$ , is to construct a correlation between the extensions of corresponding concepts such that the “remainder” is exactly 1 — which has been defined.

## 7

Thus the deduction of number as a consequence of the concept is accomplished. More exactly, from the triplet concept / truth / object, and from this unique formal operator that is biunivocal correspondence, number arises as the instance of pure thought, or integrally logical production; thought *must* suppose itself, in the form of a concept *susceptible* to having truth cases (thus endowed with an extension). This done, thought supposes number.

## 8

Why choose, in particular, the concept “not identical to itself” to found zero? Any concept could be chosen for which the extension is void, if there is no thinkable object that could have the property that designates this concept. One such concept is “square circle,” a concept of which Frege moreover declares that it “does not merit the evil that one says of it.”<sup>7</sup> Since it is a question of an integrally conceptual determination of number, the arbitrariness of the choice of the concept is a little embarrassing. Frege is aware of this, since he writes: “To define the zero, I could have taken any concept under which nothing falls.”<sup>8</sup> To avoid his own objection, Frege invokes Leibniz, whose principle of identity, which presumes that every object is identical to itself, has the merit of being “purely logical.” Purely logical? We understood that it was a question of legitimating the categories of logico-mathematics (in particular number) on the single consideration of laws of pure thought. Is there not a risk of circularity, if a logical rule is straightaway required? We will

thus say that “identical to itself” must not be confounded with “equal to itself.” Certainly, equality is one of the logical, or operatory, predicates, that it is a question of founding (and particularly equality between numbers). But, if “identity” must be carefully distinguished from the logical predicate of equality, it is clear that the statement “every object is identical to itself” is not a “purely logical” statement. *It is an ontological statement.* And, as an ontological statement, it is immediately debatable: no Hegelian, for instance, would admit the universal validity of the principle of identity. For this supposed Hegelian, the extension of the concept “not identical to itself” is anything but void!

### 9

The purely *a priori* determination of a concept whose extension is certain to be empty is an impossible task without powerful prior ontological axioms. The impasse into which Frege falls is that of an uncontrolled doctrine of the object. Because, with regard to the pure concept, what is an “object” in general, an object grasped as an arbitrary object of the total universe of objects? And why is it required that the object be identical to itself, when it is not even required of the concept that it be non-contradictory to be legitimate — as Frege indicates by affirming concepts like “square circle,” which he considers concepts just like others? Why would the law of the being of objects be more restrictive than the law of the being of concepts? This is undoubtedly so *if we admit the Leibnizian ontology*, for which existing objects obey *another principle* than thinkable objects: the principle of sufficient reason. It thus appears that the deduction of number on the basis of the concept is less universal, or “purely logical,” than it is Leibnizian.

### 10

To presume that the extension of a concept is one thing or another (for example, that the extension of the concept “not identical to itself” is empty) amounts to supposing that we can move without inconvenience from the concept to existence, since the extension of a concept puts into play “objects” that fall under this concept. There is a generalized ontological argument here, and it is this argument that sustains the deduction of number on the basis of a single concept: number belongs to the concept *in the mediation of thinkable objects that fall under the concept.*

### 11

Russell’s paradox, communicated to Frege in 1903, challenges any pretension to legislate over existence on the basis of the concept alone, and especially over the existence of the extension of concepts. Russell in effect presents a concept (in Frege’s sense), the concept “being a set that is not an element of itself” — which is certainly a completely acceptable

concept (better still, in fact, than “not identical to itself”) — but one for which, nonetheless, *the extension does not exist*. It is in effect contradictory to suppose that “objects,” in the instance of sets that “fall under this concept,” themselves form a set. And, if they do not form a set,<sup>9</sup> we cannot define some biunivocal correspondence for them. Thus this “extension” does not support equinumerosity, and consequently no number belongs to the concept “a set that is not an element of itself.”

The advent of the concept of an in-numerable ruins Frege’s general deduction. And, since the paradoxical concept in question is utterly ordinary (in fact, all the sets dealt with by mathematicians verify this concept: they are not elements of themselves), we could suspect that there exist many other concepts to which no number belongs. In fact, it is *a priori* impossible to predict the extent of the disaster. Even the concept “not identical to itself” could just as well turn out to have no existing extension, which is an entirely different thing from having an empty extension. Let us add that Russell’s paradox is purely logical, that is, precisely demonstrated: to admit the existence of a set of all those sets that are not elements of themselves ruins deductive language by introducing a *formal* contradiction (the equivalence of a proposition and its negation).

## 12

A sort of reparation has been proposed by Zermelo,<sup>10</sup> who argues that one can move from the concept to the existence of its extension *under the condition that one is operating in an already-given existence*. Given a concept C and a domain of existing objects, you can say that there exists, *in this existing domain*, the set of objects that fall under this concept, and thus the extension of this concept. This extension is evidently relative to an already specified domain of objects that does not exist “in itself.” This is a major ontological change: it is impossible in this new frame to move from the concept to existence (thus to number), but only to an existence in some way cut out from its pre-given existence. You can “separate” in a given domain the objects of this domain that validate the property exposed by the concept. This is why Zermelo’s principle, which drastically limits the rights of the concept and of language over existence, is called the axiom of separation. It even seems that, under the condition of this axiom, we are protected from the inconsistency of paradoxes such as Russell’s.

## 13

Russell’s paradox is in no way paradoxical. It is a materialist argument, because it demonstrates that being-multiple is anterior to the statements that affect it. It is impossible, says the “paradox,” to accord to language and to the concept the right to legislate without limit

over existence. Even supposing that there is a transcendental function of language is to suppose that the existent is already available, and that the power of this function only cuts out, or delimits, in this specified existent, the extensions of the concept.

#### 14

Can we, by assuming Zermelo's axiom, save the Fregean construction of number? The entire point again depends upon the question of zero. I could proceed thus: given a delimited domain of objects, whose existence is guaranteed elsewhere, I will call "zero" (or the empty set, which is the same thing) that which detaches, or separates, in this domain, the concept "not identical to itself," or any other concept such that I can assure myself that no objects of the domain fall under it. As it is a question of the limited domain, and not as in Frege's construction of "all objects" (which leads to the impasse of a Leibnizian choice without criteria), I have the chance to locate such a concept. If, for example, I take a set of black objects, I call "zero" that which separates the concept "being white." The remainder of the construction follows.

#### 15

But what domain of objects can I begin with of which it is guaranteed that they are treated by pure thought, that they are "purely logical"? Frege knew well enough to construct a concept of number that was, according to his own expression, "neither a sensible being, nor a property of external things," and he emphasized on several occasions that number is subtracted from the representable. Establishing that number is a production of thought, deducing it only from the abstract attributes of the concept in general, cannot accommodate white or black objects. The question thus becomes "of what existent can I assure myself outside of any experience?" Is the axiom "there exists something" an axiom of pure thought, and, supposing that it is, what property can I discern of which it is certain that it belongs to nothing of this existent "something"?

#### 16

As a "purely logical" demonstration of existence for the thought of an arbitrary object, of a point of Being, of an "object=x," the statement "all x is equal to itself" is an axiom of logic under equality. But the universal rules of first-order logic, a logic valid for the whole domain of objects, permit the deduction, from the statement "all x is equal to x," of the statement "there exists x that is equal to x" (subordination of the existential qualifier to the universal quantifier).<sup>11</sup> Thus, there exists x (by knowing, at least, this x that is equal to itself).

One can thus, in the frame of set theory, demonstrate *first of all*, by purely logical means, that a set exists. And *then* one can distinguish the empty set in this existent of which one has proof that it exists, by utilizing a property that no element can validate (for example, “not being equal to itself”). We have respected Zermelo’s axiom, since we have operated in a pre-given existent, and we have even engendered the zero.

### 17

I believe that this “demonstration” is an unconvincing artifice, a logical sleight of hand. Of that thing, which one universally presumes is equal to itself (which is eventually accepted as an abstract law, or law of the concept), who can reasonably infer that there *exists* something rather nothing? If the universe were absolutely void, it would remain logically admissible that, if a thing exists (which would not be the case), it would be restricted to being equal to itself. The statement “all x is equal to x” would be valid, but there would not be any x. Thus the statement “there exists x equal to itself” would not be valid.

The passage from a universal statement to an assertion of existence is an exorbitant right that the concept cannot arrogate to itself. It is not possible to establish existence on the basis of a universal law that can sustain itself just as well in absolute nothingness (see, for example, the statement “nothing is identical to itself”). And, as no existent object is deducible from pure thought, you cannot distinguish the zero there. Zermelo does not save Frege.

### 18

The existence of zero, or the empty set, and thus the evidence of numbers cannot in any way be deduced from the concept, or from language. “Zero exists” is inevitably a primary assertion, even one that fixes an existence from which all others will proceed. Far from Zermelo’s axiom, combined with Frege’s logicism, allowing us to engender the zero and then the chain of numbers, it is on the contrary the absolutely inaugural existence of zero (as the empty set) that ensures the possibility of separating some extension of the concept from what is. Number here is primary: it is this point of Being on which the exercise of the concept depends. Number, as number of nothing, or zero, sutures every text to its latent being. The void is not a production of thought, because it is from its existence that thought proceeds, inasmuch as “thinking and being are the same.”<sup>12</sup> In this sense, it is the concept that comes from number, and not the reverse.

### 19

Frege’s attempt is in certain regards unique: it is not a question of creating new intra-mathematical concepts (like Dedekind and Cantor), but — using only the resources of a



rigorous analysis — of elucidating what, among the possible objects of thought, singularizes those which fall under the concept of number. In this sense, my own effort follows along the same line. It only involves removing the obstacles by reframing the investigation according to renovated parameters. It is above all a question of showing that thought is not constituted by concepts and statements alone, but also by decisions that engage it in an epoch of its exercise.

## COMPLEMENTARY NOTE ON A CONTEMPORARY USAGE OF FREGE

### 1

Jacques-Alain Miller proposed, in a 1965 paper entitled “Suture” and subtitled “Elements of a Logic of the Signifier,”<sup>13</sup> a reprise of the construction of number by Frege. This text finds a certain logic of compatibility between structuralism and the Lacanian theory of the subject. I myself have periodically returned to this foundation<sup>14</sup> to think through this connection. Twenty-five years later, I follow, I still follow there (*j’y suis, j’y suis toujours*).

### 2

The question Miller addresses to Frege is: *What* functions in the series of whole natural numbers? The response to that question — a response, I would say, that is extorted by force from Frege — is, “in the process of the constitution of the series, the *function of the subject*, misrecognized, is operative.”

### 3

If we take this response seriously, it signifies that — in the last resort and in the proper mode of its misrecognition — it is the function of the subject for which Lacan’s teaching transmits the concept that constitutes, if not the essence, at least the process of engenderment (the “genesis of the progression,” says Miller) of number.

One evidently cannot neglect such a radical thesis. Radical with regard to the immediate import of Frege’s doctrine, which consecrates a particular development refuting the idea that number would be “subjective” (it is true for Frege that “subjective” means “caught in representation,” which evidently does not recover the Lacanian function of the subject). Radical with regard to my own thesis, since I hold that number is a form of being, and that, far from sustaining the function of the subject, it is on the contrary under the presupposition of number, and especially of this primary number-Being that is the void (or zero), that the function of the subject receives its bit of being.

## 4

It is not a question here of examining what this text — the first great Lacanian text that is not by Lacan — brings to the doctrine of the signifier, or by what analogy it illuminates the import of this still so little established epoch of all that the Master taught us about the snare of the subject in the effects of a chain. It is exclusively a question of addressing what Miller's text assumes and proposes *with regard to the thought of number as such*.

## 5

Miller organizes his demonstration in this way:

—To found zero, Frege (as we saw in 2.6) invokes the concept “not identical to itself.” No object falls under this concept. On this point, Miller emphasizes, even compounds, a Leibnizian reference of Frege's. In effect, if an object cannot be identical to or be exchanged with itself, then truth is entirely subverted. A statement that bears on an object A supposes, in order to be true, the invariance of A in each of the occurrences of the statement, or “each time” the statement is made. The principle “A is A” is a law of any possible truth. Reciprocally, in order that truth be saved, it is crucial that no object fall under the concept “not identical to itself” — whence the zero, which numbers the extension of such a concept.

—In this way, number is demonstrated to prove this single concept under the condition of truth. Now, this demonstration is uniquely consistent because if one could invoke, in thought, an object that is “not identical to itself,” it could only be for engendering the inscription of zero. Thus, writes Miller, “[t]he zero which is inscribed in the place of number consummates the exclusion of this object.”<sup>15</sup>

To say that “no object” falls under the concept “not identical to itself” is to make that object vanish through that invocation, in this nothing for which the single subsistent trait will be, precisely, the mark zero. “Our purpose,” Miller concludes, “has been to recognize in the zero-number the suturing stand in for the lack.”<sup>16</sup>

—What is it that comes to be lacking in this way? What “object” can have as the holding-place for its own default the first numerical mark, and sustain, with regard to the complete chain of numbers, the uninscribable place that pertains to its vanishing? What insists between numbers? It is necessary to agree that no

“object” can fall, even if it fails there, into the empty place that assigns non-identity-with-self. But “exist” (or here, particularly, “ek-sist”) is exactly what that which is not an object does — the non-object as such, the object as impossibility of the object, that is, the subject. “The impossible object, which the discourse of logic summons as the not-identical-with-itself and then rejects, wanting to know nothing of it, we name the object, insofar as it functions as the excess which operates in the series of numbers, the subject.”<sup>17</sup>

One must finally distinguish what Miller assumes of Frege and what can really be attributed to Frege. I will proceed in three stages.

FIRST STAGE. Miller takes as his point of departure the Leibniz-Frege proposition according to which the “salvation of truth” demands that all objects be identical to themselves. This is in fact to assume surreptitiously the integral literalization of the real to which Leibniz devoted his whole life, and of which Frege’s ideography is no doubt the inheritor. In this regard, Miller is even right to follow Leibniz and make equivalent “identical-to-itself” and “substitutability,” marking them in such a way that the letter and the object are equivalent. What can, in effect, the substitutability of an object mean? Only the letter is integrally substitutable for itself. “A is A” is a principle of letters, not objects. To be identifiable at one remove from itself, and to be submitted to a question of substitutability, is to say, in this instance, an object must be under the authority of a letter, which only calculus supposes. If A is not at all moments identical to A then truth, or veracity, as *calculus*, is annihilated.

The latent hypothesis is thus that *truth is of the order of calculus*. It is only under this supposition that it is necessary to represent the object as a letter and, as a result, the non-identical status of an object-letter radically subverts truth. And if truth is of the order of calculus, then zero — which numbers the exclusion of non-identity to self (the subject) — is only itself a letter, the letter 0. It is henceforth easy to conclude that zero is the inert holding-place of the lack, and that which “pushes” the engendering of marks in the series of numbers, a repetition where the misrecognition of what insists is spoken, is the function of the subject.

More simply, if truth is saved only through maintaining the principle of identity, the object emerges within the field of truth only as a letter offered to calculus. And, if this is the case, number can only be sustained as the repetition of that which insists as lack, which is forcibly the non-object (or the non-letter, which is the same), the place where “nothing can be written”<sup>18</sup> — in short, the subject.

Nothing is retained of Leibnizian being, although he fails to recognize in this philosophy the archetype of one of the three great orientations of thought: the constructivist or

nominalist orientation (the other two being the transcendent and the generic).<sup>19</sup> Holding the generic orientation, I declare that, for truth to be saved, it is precisely necessary to abolish the two great maxims of Leibnizian thought, which are the principle of non-contradiction and the principle of indiscernibles.

## 9

A truth supposes that the situation for which it is true occurs as “not identical to itself.” This non-identity-with-self is itself an indication that the situation has been supplemented by one multiple “too many” — for which, however, belonging or not-belonging to the situation is intrinsically undecidable. I have called this supplement “the event” and the event is always the origin of the process of a truth. Now the situation, from the time that the undecidable dice-throw of the event must be decided, necessarily enters into the wavering of its identity.

## 10

The process of a truth — making holes in the strata of knowledge that the situation holds onto — inscribes itself as indiscernible infinity, which no treasure of established language can designate.

It is enough to say that zero, or the void, has nothing in itself to do with the salvation (*salut*)<sup>20</sup> of truth, which plays in the correlative “work” between the undecidability of the event and the indiscernability of what results in the situation. It is no longer possible to refer truth to the power of the letter, since no statement can attest to the existence of a truth. The statement “truth is” — far from wagering that no object falls under the concept of “not identical to itself” and that zero is thus the number of this concept — permits this triple conclusion:

- there exists an object that occurs as “not identical to itself” (the undecidability of the event)
- an infinity of objects exist which do not fall under any concept (the indiscernability of a truth)
- number is not a category of truth

## 11

SECOND STAGE. What is the strategy of Miller’s text and what role do numbers play in it? Is it truly a matter of maintaining that the function of the subject is implicated, in the

position of misrecognized foundation, in the essence of number? This is undoubtedly what the already-cited formula clearly states: “In the process of the constitution of the series of numbers, in the genesis of progression, the function of the subject, misrecognized, is operative.”<sup>21</sup> More precisely, the function of the subject, which is inscribed in the place of lack that the zero as number marks as the place of its revocation, is alone capable of explicating what, in the series of numbers, functions as iteration, or repetition: as excluded, the subject (the “not identical to itself”) is included through the insistence of marks, incessantly to repeat the “step more” of zero approaching one (“the counting for one,” Miller notes). Then indefinitely, from  $n$  to  $n+1$ : “its exclusion [the subject in Lacan’s sense] from the field of number is assimilable to repetition.”<sup>22</sup>

## 12

Other passages of Miller’s text are more equivocal, turning into an analogical indication. For example: “if the series of numbers, metonymy of the zero, begins with metaphor, if the zero member of the series is only standing-in-place suturing the absence (of the absolute zero) which moves beneath the chain according to the alternation of a repetition and an exclusion—then what is there to prevent us from recognizing in the restored relation of the zero to the series of numbers the most elementary relation of the subject to the signifying chain?”<sup>23</sup> The word “recognizing” is compatible with the idea that the Fregean doctrine of number proposes a “matrix” (the title of another article by Miller on the same question)<sup>24</sup> that is isomorphic (maximum case), or similar (minimum case) but in any case not identical to the relation of the subject to the signifying chain. The doctrine of Frege would be an analogue (*analogon*) pertinent to Lacanian logic, and one for which we have nothing to say since, in this case, Miller’s text would not be a text on number. It would not be on two counts: initially, because it relates not to number but rather Frege’s doctrine of number (without giving a position on the validity or consistency of that doctrine), and also because it proposes the series of number as a didactic vector for the logic of the signifier, and not as an effective example of the function of the subject’s implication in the series of number.

## 13

This critical avoidance supposes that two conditions are met: first, that there is, between the doctrine of the number and that of meaning, isomorphism or similarity, and not identity or exemplification; second, that Miller does not account for the validity of the Fregean doctrine of number.

**14**

On this last point, in my view (I mean, for one who is concerned with the thought of number as such) all is suspended; Miller maintains this suspense from beginning to end. He'll mention "Frege's system" without deciding whether or not — in his view — it is the case that the theory of number is realized, a theory entirely defensible in its essence. It is striking that at no point in this very subtle and intricate exercise are the immanent difficulties of "Frege's system" ever mentioned — particularly those that I have highlighted with regard to the zero, the impact of Russell's paradox, Zermelo's axiom and ultimately, the exchange between language and existence. It thus remains possible to believe that the isomorphism between signifier and number operates between Lacan, on one hand, and, on the other hand, a version of Frege reduced to a singular theory, whose inconsistency has no impact with regard to the analogical objectives.

**15**

Evidently, it remains to be seen if this inconsistency isn't, as a result, transferred to the other pole of the analogy — thus to the logic of the signifier. The risk is not inconsequential if one imagines that the letter is placed, by Miller, in a fundamental position with regard to a very rushed logic, which the doctrine of Frege serves. "The first [the logic of the signifier] treats of the emergence of the second [logician's logic], and should be conceived of as the logic of the origin of logic."<sup>25</sup> What happens if the process of organization is completed through the theme of the subject of a schema (Frege's) struck with inconsistency? The problem is not my own. Under the conditions that I give, if the text is not regulated under number, we are free.

**16**

THIRD STAGE. There remains, however, an incontestable point of adherence on Miller's part to a general representation of number, which is taken in some way as intuitive, and with which I cannot agree. It is the idea — central, since it is precisely there that the subject makes itself known as the cause of repetition — that number is maintained in a "functioning," or in the "genesis of a progression." This is the image of a number "constructed" iteratively, on the basis of this quilting point that is the mark zero. This dynamic theme that makes number visible as passage, self-production, is omnipresent in Miller's text. The analysis centers precisely on the "passage" from zero to one, or on the "paradox of engenderment" of  $n+1$  on the basis of  $n$ .

**17**

This image of number as iteration and passage cuts out in advance any methodical discussion about the essence of number. Even if we can only cover the numerical domain according to the laws of progression (*parcourir*), for which succession is the most common (but not the only, by far), why must it follow that these laws are constitutive of the being of number? I can even see why we have to “pass” from one number to the next, or from a sequence of numbers to its limit, but it is at the very least imprudent to thereby conclude that number is defined or constituted by such passages. It might well be (and this is my thesis) that number does not pass, that it is immemorally deployed in a swarm coextensive to its being. And we will see, in the same way that we account, through difficult passages, for our journey only in its deployment, it is also likely that to this day we ignore, have no use for, or no passage onto, the considerable part of what our thought can conceive of existing numbers.

**18**

The “constructivist” thesis, that makes of iteration, of succession, of passage, the essence of number, comes to the *conclusion* that very few numbers exist, since “to exist” has no other possible sense than as the effective support of such a passage. Certainly, the intuitionists assume this impoverished perspective. Even a demi-intuitionist like Borel<sup>26</sup> thinks that the great majority of whole numbers only “exist” as a fictional and inaccessible mass. It could even be that the Leibzian choice that Miller borrows from Frege is doubled by a latent intuitionist choice.

If it is undoubtedly necessary to recognize that there is more than one common point between the logic of the signifier and intuitionist logic, this will only be because the second expressly invokes the subject (the “mathematical subject”) in its machinery. But in my opinion, such a choice would be a supplementary reason for not entering into a doctrine of number whose entire effect is that the site of number, measured by the operator intuition of a subject, is inexorably finite. For the domain of number is instead an ontological prescription incommensurable with any subject, and immersed in the infinity of infinities.

The problem thus becomes: how to think number by admitting, against Leibniz, that there are real indiscernibles; against the intuitionists, that number persists and does not pass; and against the foundational use of the subjective theme, that number exceeds all finitude?

*Translated by Sam Gillespie and Justin Clemens*

1. The selections translated here are the second and third chapters of Alain Badiou's *Le Nombre et les nombres* (Paris: Editions du Seuil, 1990).
2. ["Tout pensée émit un coup de dès." Stephen Mallarmé, "Dice Thrown Will Never Annul Chance," in *Selected Prose and Poetry*, trans. Mary Ann Caws (New York: New Directions, 1982), 127. Trans.]
3. The reference for Frege's conception of number is *The Foundations of Arithmetic*, trans. Claude Imbert (Paris: Éditions du Seuil, 1969). The first German edition is from 1884. The extremely dense fundamental development occurs in paragraphs 55-86. I [Badiou] must commend the excellent work of Claude Imbert, in particular, his long introduction. [The English translation is by J. L. Austin. See Gottlob Frege, *The Foundations of Arithmetic* (Evanston: Northwestern University Press, 1969). I have avoided the colloquialism of Austin's translation so as to stay true to Badiou's French. I will, however, footnote citations to the English text throughout. Trans.]
4. [In French, *équivalences*. Austin uses the more familiar "equal." I will use equinumerous here to emphasize that it is a purely numerical equality that is in question. Trans.]
5. ["The number which belongs to the concept F is the extension of the concept 'equal to the concept F.'" Frege, *Foundations*, 79-80. Trans.]
6. ["Our definition of the number 1 does not presuppose, for its objective legitimacy, any matter of observed fact." Frege, *Foundations*, 90.]
7. ["*Ne mérite pas le mal qu'on en dit.*" Austin translates the passage in reference to the concepts of a "wooden iron" and "square circle": "Now I believe those old friends are not so black as they are painted." Frege, *Foundations*, 87.]
8. ["I could have used for the definition of naught any other concept under which no object falls." Frege, *Foundations*, 88.]
9. The letter in which Russell communicated the paradox of his name, originally written in German, has been translated into English in *From Frege to Gödel: A Sourcebook in Mathematical Logic*, ed. Jean Van Heijenoort (Cambridge: Harvard UP, 1967), 124-5. Russell concludes by making an informed distinction between "set" (*ensemble*) and "totality." "From this I conclude that under certain circumstances, a set does not form a totality" (125). [In the English, "definable collection" is used, which Badiou then translates into French as *ensemble*, which is usually translated as "set." I have thus modified the translation of Russell. The German word in Russell's original is *Menge*. Trans.]
10. Zermelo developed his axiomatic of set theory, including the axiom of separation that reverses Russell's paradox, in a text written in 1908. [The English translation, "Investigations in the Foundation of Set Theory I" can be found in Van Heijenoort, *From Frege to Gödel*, 199-223, esp. §1: "Fundamental Definitions and Axioms," 201-205. Trans.]
11. The subordination of the existential qualifier to the universal qualifier signifies that given a property P, if all x's possess that property, then there is an x that possesses it. As given in predicate calculus:  $Ax(Px) \rightarrow Ex(P(x))$ . The axioms and rules of classical predicate calculus allow one to deduce this. Cited, for example, from the manual of E. Mendelson, *Introduction to Mathematical Logic* (D. Von Nostrand Company, 1964), 70-71.
12. [The quote, of course, is from Parmenides. We are here following the standard English translation as given, for example, in Heidegger's *Introduction to Metaphysics*, trans. Ralph Manheim (New Haven: Yale University Press, 1959), 136. Louise Burchill, in her translation of Badiou's *Deleuze*, uses "the same, it is thinking and being" to translate Badiou's French: "*le même, lui, est à la fois penser et l'être.*" She notes, furthermore, that Badiou's translation conforms to Jean Beaufort's French translation of *La Poème de Parménide* (Paris: PUF, 1955), 56. See Badiou, *Deleuze: The Clamor of Being*, trans. Louise Burchill (Minneapolis: University of Minnesota Press, 2000), 137. Trans.]
13. Miller's text was published in *Cahiers pour l'analyse 1* (Paris: Éditions du Seuil, 1966). It completes a lecture, in the same issue of the journal, of an article by Y. Duroux, "Psychology and Logic," which examines in detail the function of the successor in Frege. [I am here relying, with occasional modifications, on Jacqueline Rose's translation of Miller. See Jacques-Alain Miller, "Suture



(elements of a logic of the signifier)," in *Screen* 18.4 (Winter, 1977-8): 24-34. Trans.]

14. Alain Badiou, "Marque et Manque: à propos du Zero," in *Cahiers pour l'analyse* 10 (May, 1969).
15. Miller, "Suture (elements of a logic of the signifier)," 30.
16. *Ibid*, 31.
17. *Ibid*, translation modified.
18. *Ibid*, 30. Translates "...rien ne saurait être écrit."
19. For a typology of these orientations of thought, see Mediation 27 of Alain Badiou, *L'Être et l'événement* (Paris: Éditions du Seuil, 1988), 311-315.
20. ["Salut" has a religious meaning of "salvation." Trans.]
21. Miller, "Suture (elements of a logic of the signifier)," 27. [In the French, "in the genesis of progression" and "misrecognized" are deleted and replaced with ellipses.]
22. *Ibid.*, 27.
23. *Ibid.*, 32, translation modified.
24. Miller "Matrice," *Ornicar?* 4 (1975). Translated by Daniel G. Collins in *lacanian ink* 12 (Fall, 1997): 48-51.
25. Miller, "Suture (elements of a logic of the signifier)," 25.
26. See for example, E. Borel, "Mathematical Philosophy and Infinity," in *Revue du mois* 14 (1912): 219-227.

